Consistent Velocity Interpolation for SIMPLE and PIMPLE Algorithm in Collocated Grid

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Previous Works…

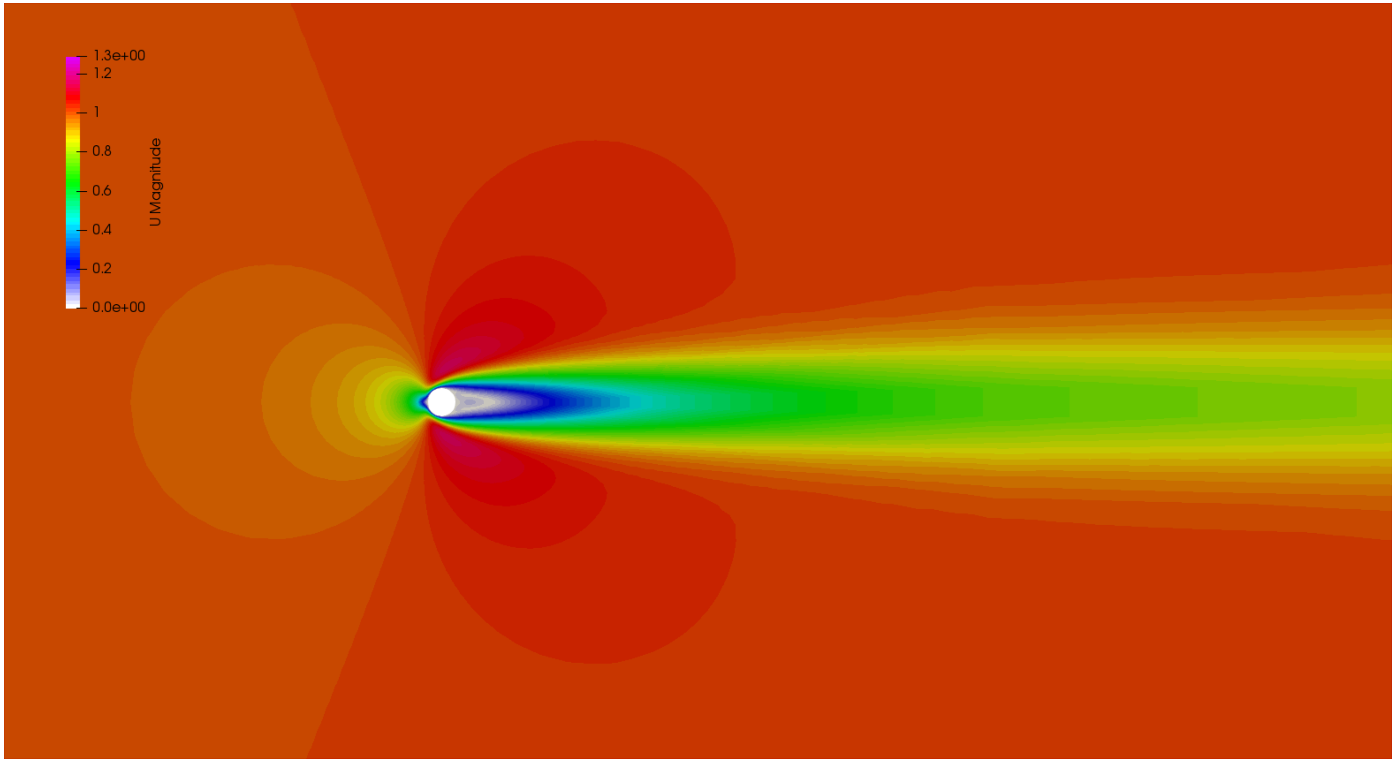
- SIMPLE 알고리즘의 pressure under-relaxation 순서
- Rhie-Chow Interpolation
- Non-orthogonal correction
- 계산 초기의 난류값 및 난류점성계수 제한
- 난류 생성항 선형화 방법
- 복합열유동해석시 유체/고체 경계면의 열전달 경계조건
- 중력에 의한 생성항 처리 방법
- Operating Pressure
- 운동량 보존방정식의 압력구배항 이산화 방법

Present Interest

- Steady, Unsteady 솔버간의 일관성
  - Steady 문제 해석시 동일한 해
  - Under-relaxation
  - Time-step size

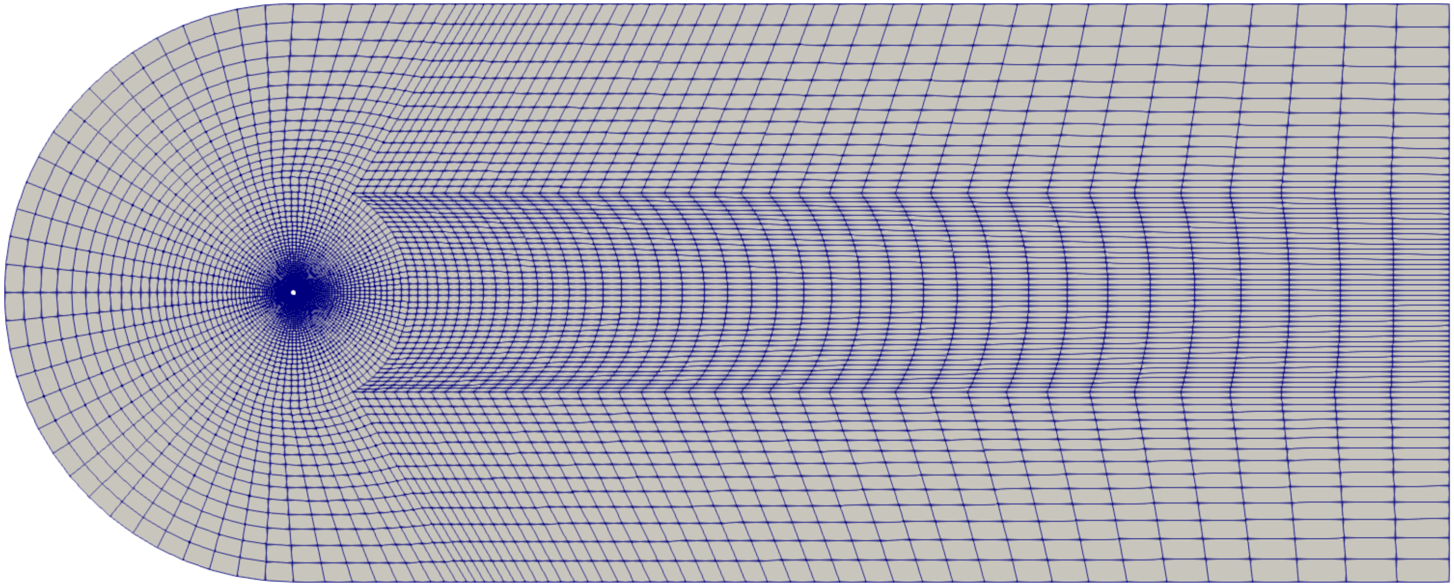
간단한 문제...

- Laminar flow around a circular cylinder (Re=25)



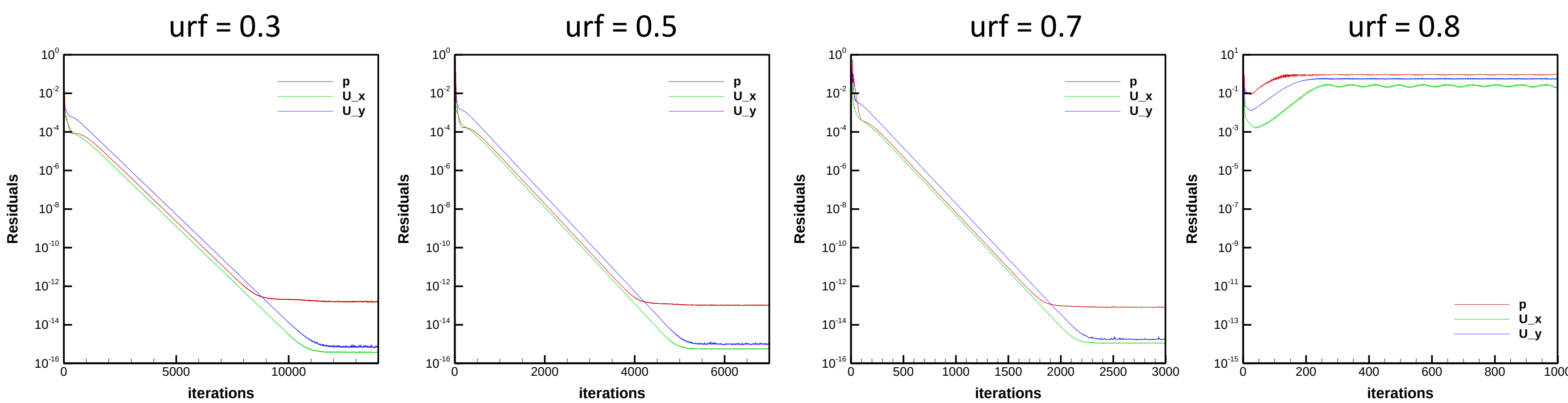
간단한 문제...

- Grid



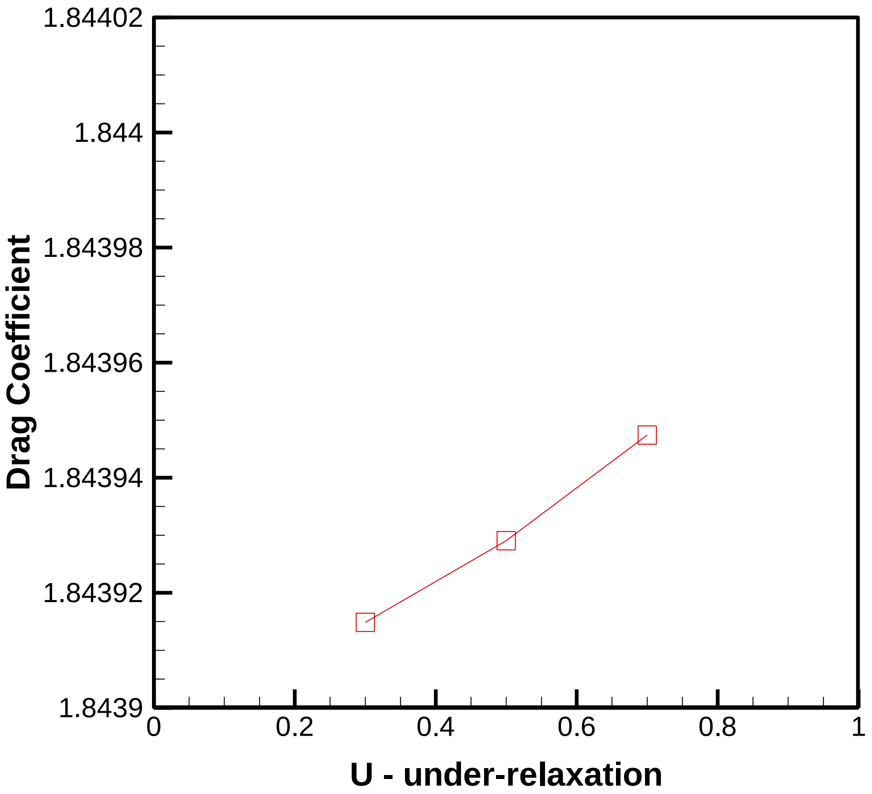
Under-relaxation factor의 영향 - simpleFoam

- 운동량 보존방정식의 under-relaxation factor에 따른 수렴성 비교



Under-relaxation factor의 영향 - simpleFoam

- 항력 계수



30년 전…

- S. Majumdar, Role of Underrelaxation in Momentum Interpolation for Calculation of Flow with Nonstaggered Grids, *Numer. Heat Transfer*, vol. 13, pp. 125-132, 1988.
- T. F. Miller and F. W. Schmidt, Use of a Pressure-Weighted Interpolation Method for the Solution of Incompressible Navier-Stokes Equations on a Non-Staggered Grid System, *Numer. Heat Transfer*, vol. 14, pp. 213-233, 1988.
- M. H. Kobayashi and J. C. F. Pereira, Numerical Comparison of Momentum Interpolation Methods and Pressure-Velocity Algorithm Using Nonstaggered Grids, *Commun. Appl. Numer. Meth.*, vol. 7, pp. 173-196, 1991.

Face Flux in `simpleFoam`

- Cell 중심에서의 속도에 대한 표현

$$\vec{U}_P = \frac{H(\vec{U})}{a_P} - \frac{V_P}{a_P}(\nabla p)_P$$

  - Cell P에 대해서 이산화한 운동량 보존방정식에서 유도

- Pseudo-Velocity

$$\hat{U}_P = \frac{H(\vec{U})}{a_P}$$

- Pseudo-Face Flux

$$\hat{\phi} = \vec{S}_f \cdot \hat{U}_f$$

Face Flux in `simpleFoam`

- How to evaluate $\hat{U}_f$
  - Interpolation

$$\hat{U}_f \approx \overline{\left(\hat{U}_P\right)}_f = \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f$$

- Then, pseudo-face flux:

$$\hat{\phi} = \vec{S}_f \cdot \hat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f$$

```
surfaceScalarField phiHbyA("phiHbyA", fvc::interpolate(HbyA) & mesh.Sf());
```

OR

```
surfaceScalarField phiHbyA("phiHbyA", fvc::flux(HbyA));
```

Consistent pseudo-face flux in `simpleFoam`

- $\frac{H(\vec{U})}{a_P}$에 포함된 항들은?

$$\frac{H(\vec{U})}{a_P} = \frac{1}{a_P}\left\{-\sum_N a_N \vec{U}_N + (1-\alpha_U) a_P \vec{U}_P^{n-1} - \sum_f \vec{U}_f^{corr} + \sum_f \left(\mu_{eff}\right)_f \left|\vec{S}_f\right| \cdot \vec{k}\left(\nabla \vec{U}\right)_f\right\}$$

  – $\sum_f \vec{U}_f^{corr}$: 대류항 이산화에 따른 생성항
  – $\sum_f \left(\mu_{eff}\right)_f \left|\vec{S}_f\right| \cdot \vec{k}\left(\nabla \vec{U}\right)_f$: 확산항 이산화에 따른 생성항

Consistent pseudo-face flux in `simpleFoam`

- Pseudo-Face Flux $\hat{\phi} = \vec{S}_f \cdot \hat{U}_f$

$$\vec{S}_f \cdot \hat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}_f}$$

$$= \vec{S}_f \cdot \overline{\left\{-\frac{1}{a_P}\sum_N a_N \vec{U}_N\right\}_f} + (1-\alpha_U)\vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)_f}$$

$$-\vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \vec{U}_f^{corr}\right\}_f} + \vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f (\mu_{eff})_f |\vec{S}_f| \cdot \vec{k}(\nabla\vec{U})_f\right\}_f}$$

Consistent pseudo-face flux in `simpleFoam`

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$$\vec{S}_f \cdot \hat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f$$

$$= \vec{S}_f \cdot \overline{\left\{-\frac{1}{a_P}\sum_N a_N \vec{U}_N\right\}}_f + (1-\alpha_U)\vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)}_f$$

$$-\vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \vec{U}_f^{corr}\right\}}_f + \vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \left(\mu_{eff}\right)_f \left|\vec{S}_f\right| \cdot \vec{k}\left(\nabla\vec{U}\right)_f\right\}}_f$$

Consistent pseudo-face flux in simpleFoam

- Pseudo-Face Flux $\hat{\phi} = \vec{S}_f \cdot \hat{U}_f$

$$\vec{S}_f \cdot \hat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}_f}$$

$$= \vec{S}_f \cdot \overline{\left\{-\frac{1}{a_P}\sum_N a_N \vec{U}_N\right\}_f} + (1-\alpha_U)\phi^{n-1}$$

$$-\vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \vec{U}_f^{corr}\right\}_f} + \vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f (\mu_{eff})_f |\vec{S}_f| \cdot \vec{k}(\nabla\vec{U})_f\right\}_f}$$

Consistent pseudo-face flux in `simpleFoam`

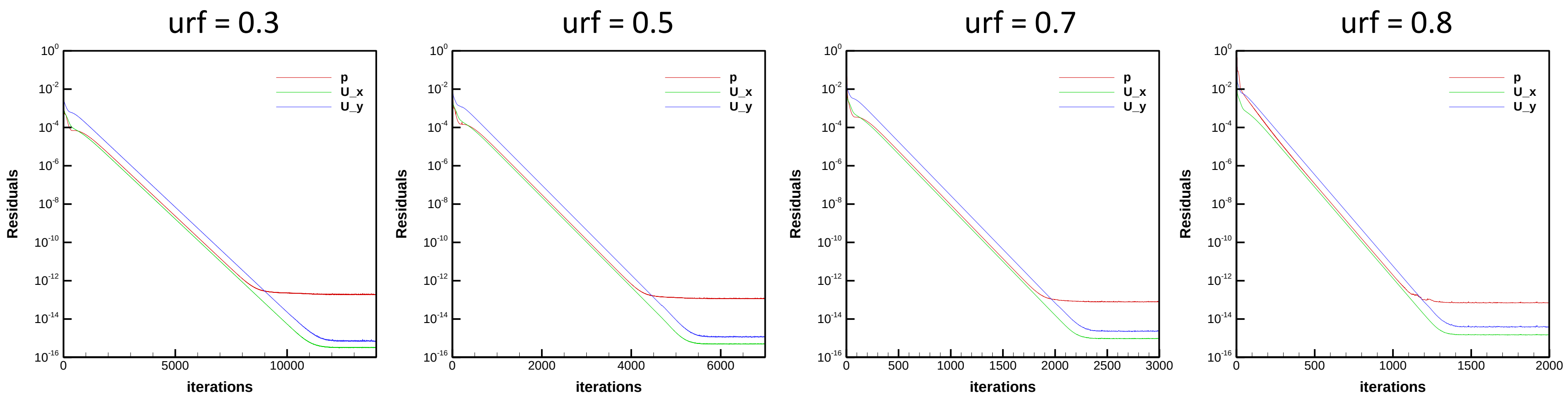
- Correcting Pseudo-Face Flux

$$\vec{S}_f \cdot \widehat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f + (1-\alpha_U)\left\{\phi^{n-1} - \vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)}_f\right\}$$

```
scalar UUrf = mesh.equationRelaxationFactor(U.name());
surfaceScalarField phiHbyA
(
    "phiHbyA",
    (fvc::interpolate(HbyA) & mesh.Sf())
  + (1.0 - UUrf)*phi.prevIter()
  - (1.0 – Uurf)*(fvc::interpolate(U.prevIter()) & mesh.Sf())
);
```

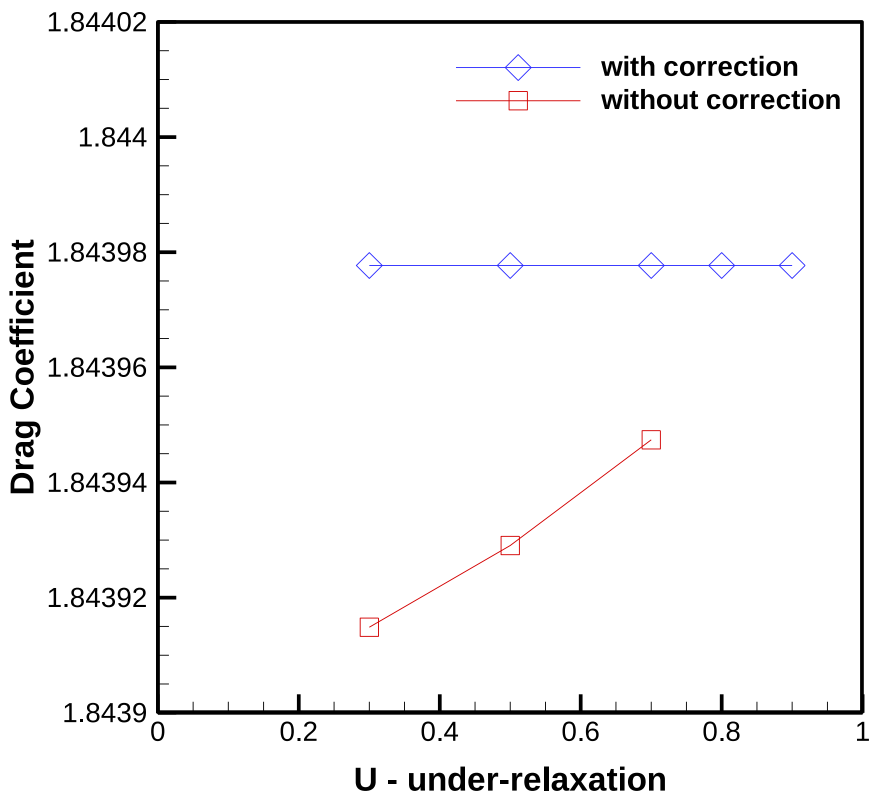
Under-relaxation factor의 영향 - simpleNFoam

- Consistent pseudo-face flux
- 운동량 보존방정식의 under-relaxation factor에 따른 수렴성 비교



Under-relaxation factor의 영향 - simpleNFoam

- 항력 계수



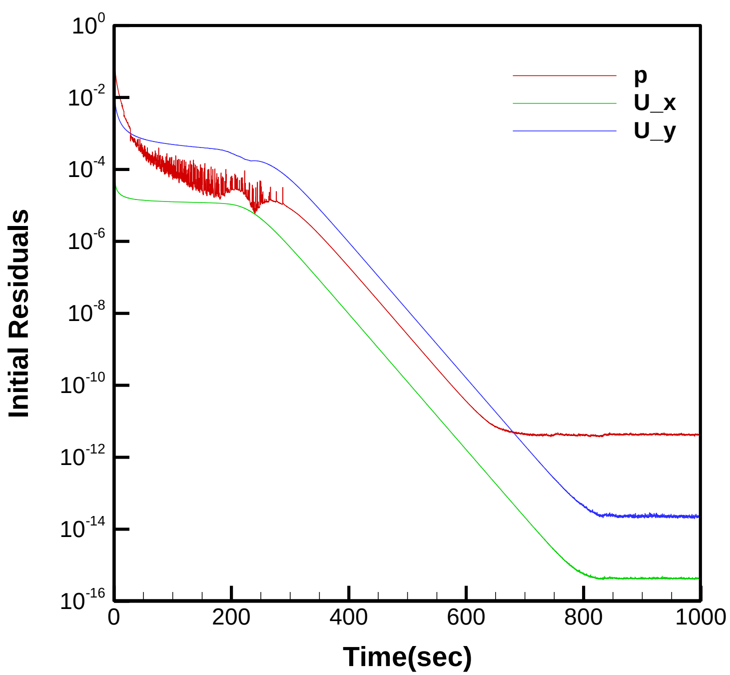
Time-step size의 영향 - pimpleFoam

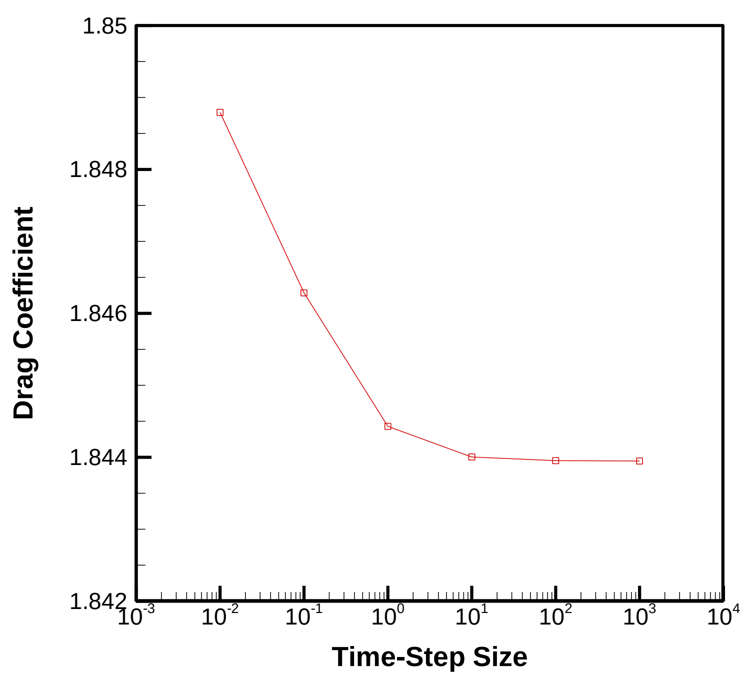
- **fvSolution**

```
PIMPLE
{
    nNonOrthogonalCorrectors    0;

    nCorrectors                 2;
    nOuterCorrectors            20;

    residualControl
    {
        p
        {
            relTol          0.0;
            tolerance       1e-4;
        }
        U
        {
            relTol          0.0;
            tolerance       1e-3;
        }
    }
}
```

Time-step size의 영향 - pimpleFoam

- ddtSchemes : Euler
- 수렴성 및 항력 계수





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- S. K. Choi, Note on the Use of Momentum Interpolation Method for Unsteady Flows, *Numer. Heat Transfer A*, vol. 36, pp. 545-550, 1999.
- B. Yu, Y. Kawaguchi, W. Q. Tao, and H. Ozoe, Checkerboard Pressure Predictions Due to the Underrelaxation Factor and Time Step Size for a Nonstaggered Grid with Momentum Interpolation Method, *Numer. Heat Transfer B*, vol. 41, no. 1, pp. 85-94, 2002.

Consistent pseudo-face flux in `pimpleFoam`

- $\frac{H(\vec{U})}{a_P}$에 포함된 항들은?

$$\frac{H(\vec{U})}{a_P}$$

$$= \frac{1}{a_P}\left\{-\sum_N a_N \vec{U}_N + (1-\alpha_U) a_P \vec{U}_P^{n-1} + \frac{V_P}{\Delta t}\vec{U}_P^o - \sum_f \vec{U}_f^{corr} + \sum_f \left(\mu_{eff}\right)_f \left|\vec{S}_f\right| \cdot \vec{k}\left(\nabla \vec{U}\right)_f\right\}$$

Consistent pseudo-face flux in `pimpleFoam`

- Pseudo-Face Flux $\hat{\phi} = \vec{S}_f \cdot \widehat{U}_f$

$$\vec{S}_f \cdot \widehat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f$$

$$= \vec{S}_f \cdot \overline{\left\{-\frac{1}{a_P}\sum_N a_N \vec{U}_N\right\}}_f + (1-\alpha_U)\vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)}_f$$

$$+\vec{S}_f \cdot \overline{\left\{\frac{1}{\Delta t}\left(\frac{V_P}{a_P}\right)\vec{U}_P^o\right\}}_f$$

$$-\vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \vec{U}_f^{corr}\right\}}_f + \vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \left(\mu_{eff}\right)_f |\vec{S}_f| \cdot \vec{k}(\nabla\vec{U})_f\right\}}_f$$

Consistent pseudo-face flux in `pimpleFoam`

- Pseudo-Face Flux $\hat{\phi} = \vec{S}_f \cdot \hat{U}_f$

$$\vec{S}_f \cdot \hat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}_f}$$

$$= \vec{S}_f \cdot \overline{\left\{-\frac{1}{a_P}\sum_N a_N \vec{U}_N\right\}_f} + (1-\alpha_U)\vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)_f}$$

$$+\vec{S}_f \cdot \overline{\left\{\frac{1}{\Delta t}\left(\frac{V_P}{a_P}\right)\vec{U}_P^o\right\}_f}$$

$$-\vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \vec{U}_f^{corr}\right\}_f} + \vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f (\mu_{eff})_f |\vec{S}_f| \cdot \vec{k}(\nabla\vec{U})_f\right\}_f}$$

Consistent pseudo-face flux in `pimpleFoam`

- Pseudo-Face Flux $\hat{\phi} = \vec{S}_f \cdot \hat{U}_f$

$$\vec{S}_f \cdot \hat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}_f}$$

$$= \vec{S}_f \cdot \overline{\left\{-\frac{1}{a_P}\sum_N a_N \vec{U}_N\right\}_f} + (1-\alpha_U)\phi^{n-1} + \overline{\left\{\frac{1}{\Delta t}\left(\frac{V_P}{a_P}\right)\right\}_f}\phi^o$$

$$-\vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f \vec{U}_f^{corr}\right\}_f} + \vec{S}_f \cdot \overline{\left\{\frac{1}{a_P}\sum_f (\mu_{eff})_f |\vec{S}_f| \cdot \vec{k}(\nabla\vec{U})_f\right\}_f}$$

Consistent pseudo-face flux in pimpleFoam

- Correcting Pseudo-Face Flux

$$\vec{S}_f \cdot \widehat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f + (1-\alpha_U)\left\{\phi^{n-1} - \vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)}_f\right\}$$

$$+ \left[\overline{\left\{\frac{1}{\Delta t}\left(\frac{V_P}{a_P}\right)\right\}}_f \phi^o - \vec{S}_f \cdot \overline{\left\{\frac{1}{\Delta t}\left(\frac{V_P}{a_P}\right)\vec{U}_P^o\right\}}_f\right]$$

Consistent pseudo-face flux in `pimpleFoam`

- **`pEqn.H`** in **`pimpleFoam`**

$$\vec{S}_f \cdot \widehat{U}_f \approx \vec{S}_f \cdot \overline{\left\{\frac{H(\vec{U})}{a_P}\right\}}_f + (1-\alpha_U)\left\{\phi^{n-1} - \vec{S}_f \cdot \overline{\left(\vec{U}_P^{n-1}\right)}_f\right\}$$

$$+\phi_{coeff}\left[\overline{\left\{\frac{1}{\Delta t}\left(\frac{V_P}{a_P}\right)\right\}}_f \phi^o - \vec{S}_f \cdot \left\{\frac{1}{\Delta t}\overline{\left(\frac{V_P}{a_P}\right)}_f \overline{\left(\vec{U}_P^o\right)}_f\right\}\right]$$

```
surfaceScalarField phiHbyA
(
    "phiHbyA",
    fvc::flux(HbyA)
  + fvc::interpolate(rAU)*fvc::ddtCorr(U, phi)
);
```

Consistent pseudo-face flux in pimpleFoam

- **EulerDdtScheme.C**

```
template<class Type>
tmp<typename EulerDdtScheme<Type>::fluxFieldType>
EulerDdtScheme<Type>::fvcDdtPhiCorr
(
    const GeometricField<Type, fvPatchField, volMesh>& U,
    const fluxFieldType& phi
)
{
    dimensionedScalar rDeltaT = 1.0/mesh().time().deltaT();

    fluxFieldType phiCorr
    (
        phi.oldTime() - fvc::dotInterpolate(mesh().Sf(), U.oldTime())
    );

    return tmp<fluxFieldType>
    (
        new fluxFieldType
        (
            IOobject
            (
                "ddtCorr(" + U.name() + ',' + phi.name() + ')',
                mesh().time().timeName(),
                mesh()
            ),
            this->fvcDdtPhiCoeff(U.oldTime(), phi.oldTime(), phiCorr)
           *rDeltaT*phiCorr
        )
    );
}
```

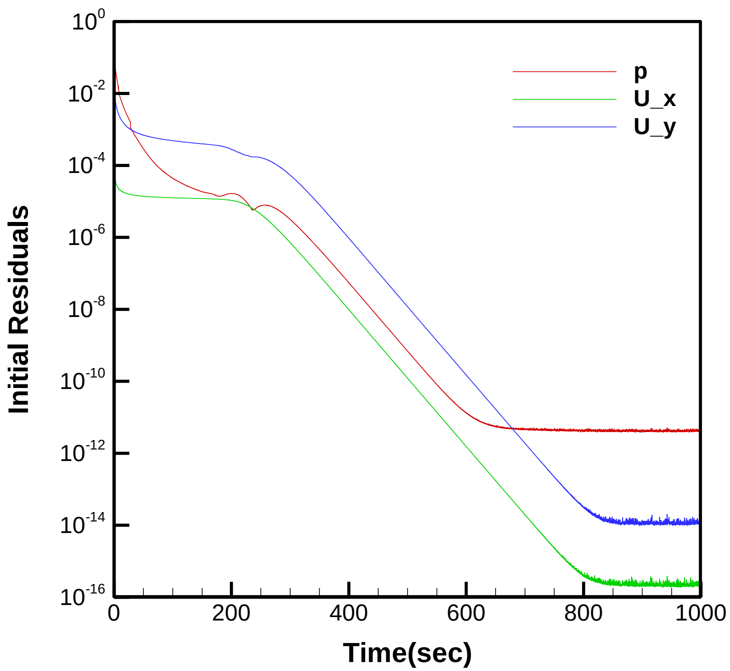
Consistent pseudo-face flux in `pimpleFoam`

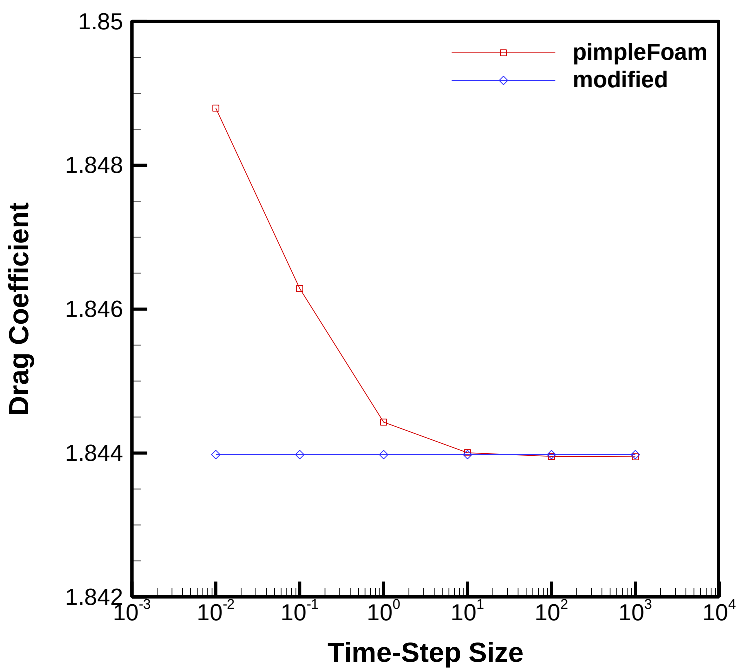
- 개발자의 실수?

$$\overline{\left\{\left(\frac{V_P}{a_P}\right)\vec{U}_P^o\right\}_f} \neq \overline{\left(\frac{V_P}{a_P}\right)_f}\,\overline{\left(\vec{U}_P^o\right)_f}$$

Time-step size의 영향 - pimpleNFoam

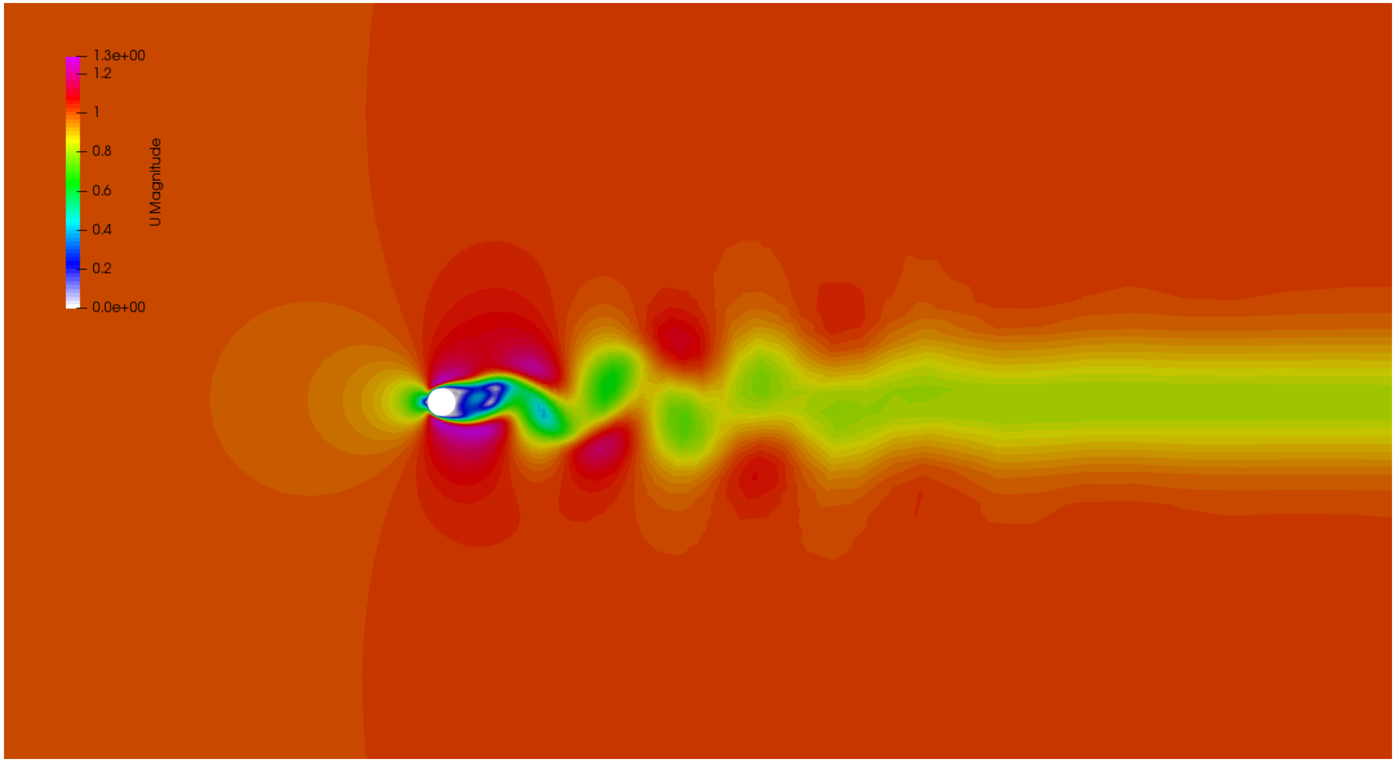
- 수렴성 및 항력 계수





pimpleNFoam 은 steady solver?

- Laminar flow around a circular cylinder (Re=100)



pimpleNFoam 은 steady solver?

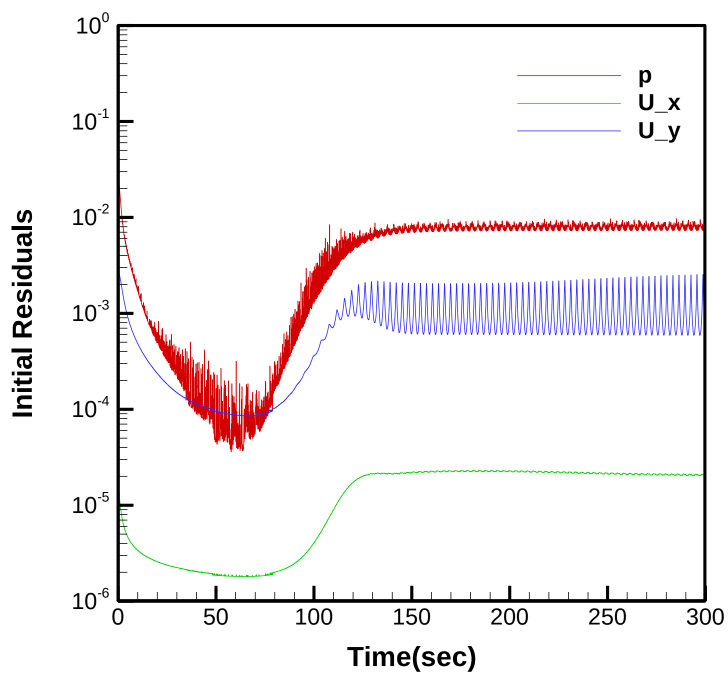
- **ddtSchemes: backward**
- **fvSolution**

```
PIMPLE
{
    nNonOrthogonalCorrectors    0;

    nCorrectors                 2;
    nOuterCorrectors            40;

    residualControl
    {
        p
        {
            relTol          0.0;
            tolerance       1e-4;
        }
        U
        {
            relTol          0.0;
            tolerance       1e-4;
        }
    }
}
```

pimpleNFoam 은 unsteady solver

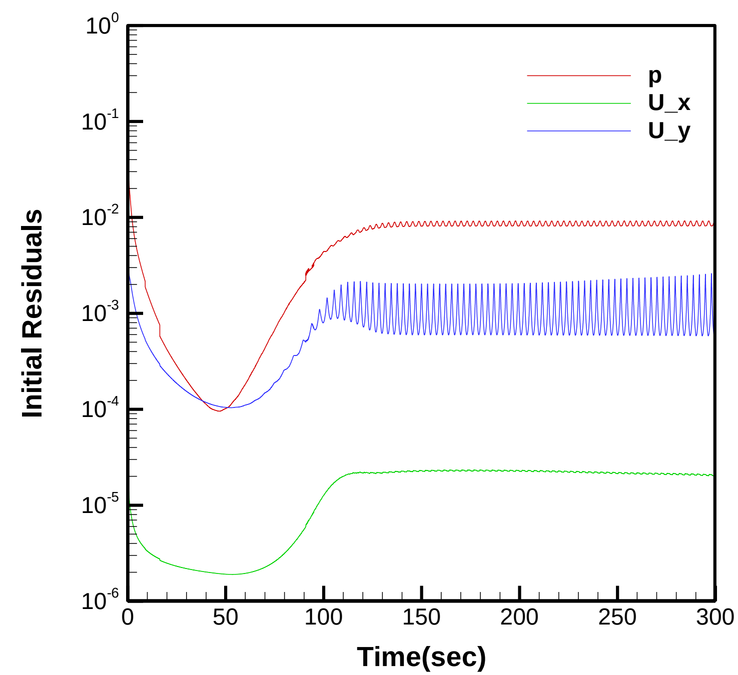
- 수렴성

pimpleFoam



13 PIMPLE itr.

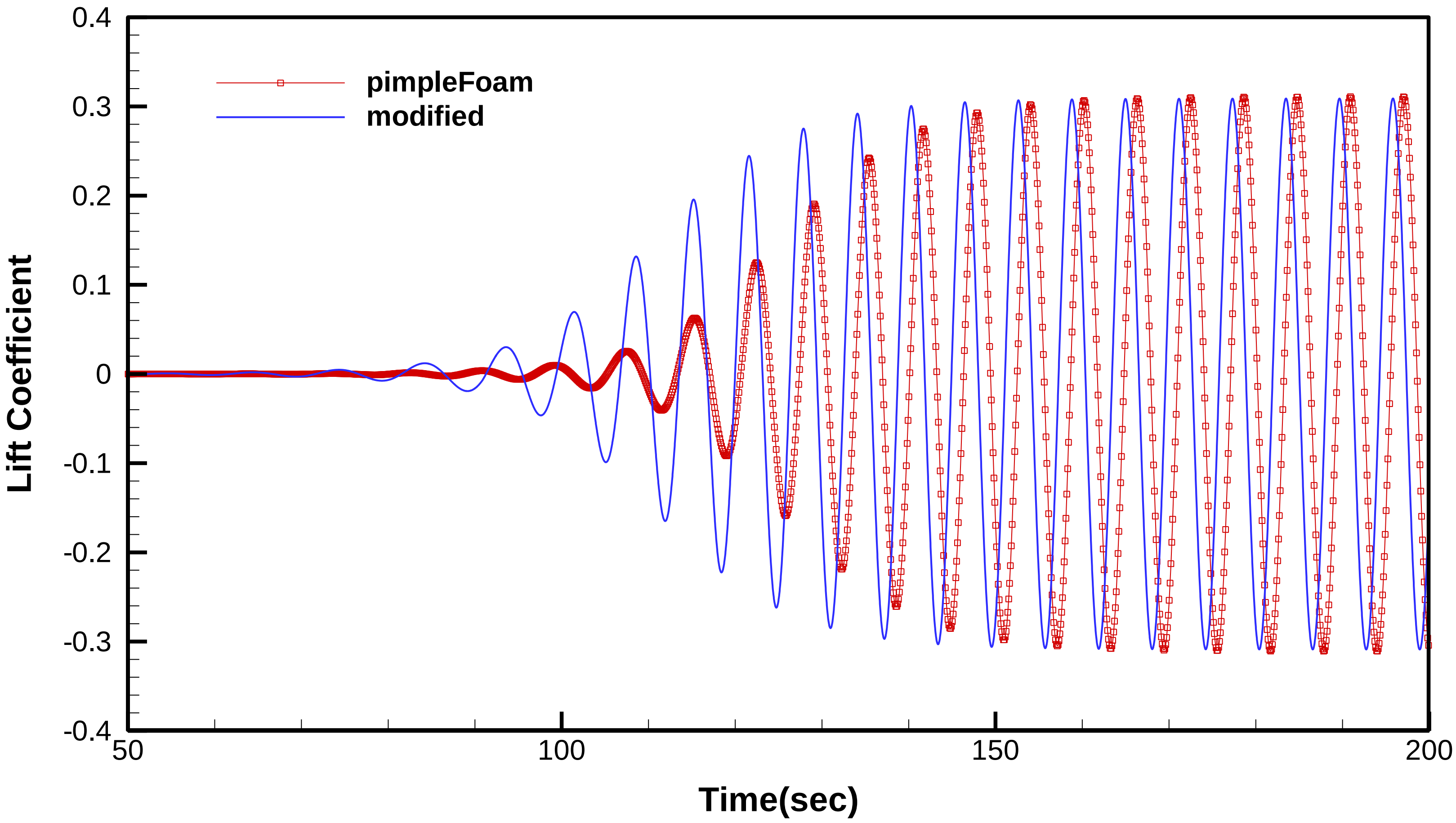
pimpleNFoam



9 PIMPLE itr.

pimpleNFoam 은 unsteady solver

- 양력 계수



Future Works

- 열전달 솔버에 적용
- Dynamic Mesh…